



# GEO - INTEGRATION

CONDUCTED RESEARCH

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## INTRODUCTORY FRAME OF THE TOPIC, AIMS & OBJECTIVES

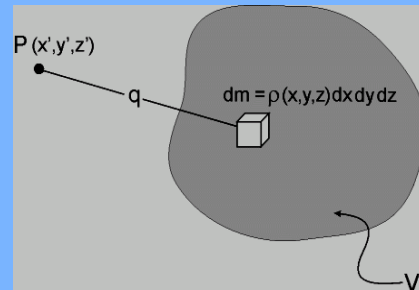
The aim of this proposal had been to contribute to the theoretical interpretation – from the Mathematical point of view – of the orthometric height system that is mostly used in Geodesy in all its mapping and positioning objectives. This height system is through Physics principally using the classic Newtonian relation:

$$g = \text{grad}(W)$$

where  $g$  is the local gravity force and  $W$  represents the gravitational potential of the Earth's natural body.

This definition provides immediately two consequences:

- 1) The first one is that the orthometric system and consequently the orthometric heights  $H$  of Geodesy are a natural coordinate defined in a space having variable norm
- 2) The determination of orthometric height of any geo-referenced point in space or on the topographic surface is possible only through a Mathematical description of the Earth's gravity field

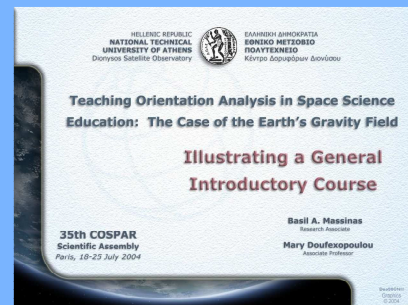


## THE MOST IMPORTANT OBJECTIVES

The aim is targeted to certain objectives – among several existing - the most important of which are:

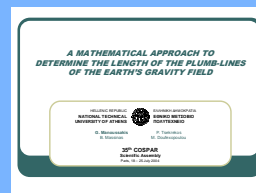
- A study to produce an analytical relation in which the length of a plumb-line is determined between two equipotential surfaces of the Earth's gravity field by using various coordinate systems well known to Geodesists
- The Mathematical classification of some basic external boundary value problems of the gravitational potential that are the basis on which rely all methods to determine the potential  $W$ . Note that the determination of the potential in the Earth's outer space and on topography is necessary for all Earth and Space positioning applications of Geodesy
- An educational study how to approach the teaching subject of the Earth's gravity field to learners coming from a wide variety of background knowledge and need a basic subject course on this topic for quite various objectives. The used criteria are the simplicity of the subject's presentation and the initiation of the interest of the audience.

A by-product of the previous objectives has been the extension of the teaching approach from the Earth's gravity field into the whole subject of Geodesy. As it is well known to Geodesists Geodesy is characterised by duality between Physics and Geometry, a feature mostly illustrated by the orthometric and the geometric height systems that bear different definitions and do have important differences. The difference between the two systems can be expressed only by the Earth's gravity field modelling on its surface. Although Geodesy is understood as an applied discipline of Earth sciences, it is strongly founded on Physics and Mathematics.



### Gravity: Space as a Rubber Sheet

- Matter tells space how to curve
- Curved space tells matter how to move



### The Integral approach

The length of the plumb-line is given by the formula

$$H_r = \int \left[ \left( \frac{d^2x}{dt^2} \right)^2 + \left( \frac{d^2y}{dt^2} \right)^2 + \left( \frac{d^2z}{dt^2} \right)^2 \right]^{1/2} dt$$

and if  $H_r^*$  is the real length of the plumb-line then it holds that

$$\left| H_r^* - H_r \right| < \epsilon \sum_{i=1}^n \epsilon_i, \quad 0 < \epsilon < 10^{-7}$$

## THE PROJECT'S "PRODUCTS"

(1) The first mentioned objective has been the main target of research conducted by the post graduate student of Rural & Surveying Eng., Dipl. Eng. Mr. G. Manoussakis, who devoted considerable time to conceive the Physical laws behind the height system. Mr Manoussakis tried on several global approaches to define the kernel of integration of the plumb-lines. He determined an approach to define the length of plumb-lines under the criterion of closed formulae and he used two "classic" Mathematical methods:

- a) Approach with integration
- b) A differential geometry approach

Part of these attempts is to be included in his doctoral dissertation under preparation. A concise presentation of the two methods using a global spherical coordinate system and assuming that the potential is given in a spherical harmonic expansion, has been presented in oral poster form at the 35th General Assembly of COSPAR on July 2004 under the title:

"A Mathematical Approach to determine the length of the plumb-lines of the Earth's Gravity field"

(G. Manoussakis, B. Massinas, P. Tsekrekos, M. Doufexopoulou)

(The paper accepted for publication by Elsevier in the 'Advances in Space Research').

(2) The second objective has been accomplished by an elaborated unpublished paper done by the project's coordinator Associate prof. M. G. Doufexopoulou. The elaboration is based on two lengthy unpublished and detailed descriptions of some classic external boundary value problems prepared by Mr. Manoussakis during the present project. The classification of the basic boundary value problems is given in form of three tables (The text is distributed free) and is already used as optional learning material from students of the Rural & Surveying Eng. Faculty. It is planned in the future to build e-learning self educating material in form of ed.

(3) The third objective has been rather fruitful. Actually the coordinator M. Doufexopoulou and Mr. B. Massinas, PhD candidate of the Faculty went through extended theoretical and experimental bibliographical research on didactical approaches. Also, the coordinator has used collected experimental experience on educational approaches and the result of these activities was implemented for the task to define ways to trigger the motivation and the interest of students to the teaching subject of "The Earth's gravity field". A first "product" was the paper presented in the educational session of the 35th general assembly of COSPAR:

"Teaching orientation analysis in space science education. The case of the Earth's gravity field" (B. Massinas, M. Doufexopoulou) (The paper accepted for publication by Elsevier in the 'Advances in Space Research').

The paper combines basic didactical rules, the duality of gravity space in concept of geometry and as Mathematically described by Newtonian theory and uses the optical representation of the gravitational potential energy variation through different equipotential surfaces.

Last but not least the frame of this project's research initiated further collaborations either in experimental research through a paper presented in Jaen, Spain March 2005 (submitted for publication in Springer).

### PUBLICATIONS

1) "A Mathematical Approach to determine the length of the plumb-lines of the Earth's Gravity field"

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3) "Spectral Analysis of Geoidal Signals at points of geodynamical interest used in the investigation of the depth of mass-density causal 'sources' of ground deformations" (M. Doufexopoulou, G. Bartha, B. Massinas) (submitted for publication in Springer).