Investigation of Mathematical approaches leading to convert lengths between the natural and mathematical space that embeds the corresponding Earth models

Research team

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Introduction

The aim of this project was the contribution to a theoretical interpretation – in Mathematical terms - of the orthometric height system mostly used in Geodesy in all mapping and positioning objectives. This aim includes two main objectives: The first one was a Mathematical research to express the length of plumb-lines, defined in the gravity space, via geodetic coordinates (defined in a Cartesian space). The second objective was oriented to approaches in teaching this particular subject in undergraduate and postgraduate level, in such a form that the apparently lengthy Mathematical formulation could be well reasoned for its necessity in dealing with height problems in Earth disciplines. Particularly it was planed to prepare a Mathematical classification of related geodetic boundary value problems of the gravitational potential of the Earth in form of educational material for post-graduates in Earth disciplines.

The height systems used in Earth disciplines

Geodesy and other Earth disciplines use in parallel today two different height systems, one defined through Physics using the classic Newtonian relation:

g=grad(W)

where **g** is the local gravity force and **W** represents the gravitational potential of the Earth's natural body (called orthometric) and one defined by a corresponding relation

γ=grad(U)

where γ the gravity force on an the Earth's dynamical ellipsoidal model which is defined in its geometrical and dynamical features that produce the normal potential U (called geometric system). The orthometric system is mostly used because height differences in this system define the natural direction of flow, so it is called "environmental" height. However satellite based positioning methods, such as GPS provide heights defined in an equivalent to the geometric

height system that is based also on time (WGS 84). The conversion between the two types of heights involves the determination of the gravitational potential W at points of height conversion (through methods coming from the solution of particular external boundary value problems of the potential) and by neglecting the difference between the direction of the vectors g and γ (a small but not negligible angle, called deflection of the vertical). The project's main objective was targeted by the researcher G. Manoussakis to the goal:

To produce an analytical relation in which the length of a plum-line is determined between two equipotential surfaces of the Earth's gravity field by using various coordinate systems used by Geodesists. The second objective defined by the project's coordinators was the Mathematical classification of some basic external boundary value problems of the gravitational potential that operate as the theoretical arsenal of the used methods to determine the potential W. The determination of the potential W in the Earth's outer space and on topography in needed in all Earth and Space positioning applications related to Earth and Space Sciences. Consequently a counterpart objective set up in the course of this research has been to conduct an educational study how to approach the teaching subject of the Earth's gravity field addressed to learners coming from a wide variety of background knowledge and need a basic subject course on this topic for quite various objectives. The used criteria are from the field of didactics as 1) simplicity of the subject's presentation and 2) initializing the interest of a broad learning audience.

As it is well known to Geodesists, Geodesy is characterised by duality between Physics and Geometry, a feature mostly illustrated by the two height systems defined in the gravity and geometry space. The difference between the two systems is illustrated through the Earth's gravity field modelling via the harmonic difference function T=W-U in space where no matter exists. Although Geodesy is standing as an applied discipline in Earth's engineering, it has strong foundations on Physics and Mathematics.

Summarizing the project's "products"

The first objective that was the principal target of research was conducted by Dipl. Eng. Mr. G. Manoussakis, under guidance of P. Tsekrekos for Mathematics and coordination of M. Doufexopoulou for its geodetic relevance. G. Manoussakis devoted considerable time to conceive the Physical problem behind the height system. Particularly he tried on several global approaches to define the kernel of integration of the plumb-lines and he determined an approach to define the length of plumb-lines under the criterion of closed formulae, This task concluded to apply two "classic" Mathematical methods:

a) Approach with integration
b) A differential geometry approach

A brief presentation of the two approaches using a global spherical coordinate system and assuming that the potential is given in a spherical harmonic expansion, has been presented in oral poster form at the 35th General Assembly of COSPAR on July 2004, titled:

A Mathematical Approach to determine the length of the plumb-lines of the Earth's Gravity field
(G. Manoussakis, B. Massinas, P. Tsekrekos, M. Doufexopoulou).

The second objective is in form of a technical report was implemented by the project's coordinator Associate prof. M. G. Doufexopoulou with the rest of the research team. The report

used two lengthy unpublished and detailed descriptions on some classic external boundary value problems prepared by Mr. Manousakis under guidance of P. Tsekerekos. This initial material has been fully re-built under the two main approaches of geodetic boundary value problems namely the Stokes (1849) and the Molodensky (1962) approach. The classification of some boundary value problems met in Geodesy has been done under the type of the Mathematical solution and is presented in three tables. The report is available from mairiedf@central.ntua.gr for academic use. The report is already used as optional learning material from students of the Rural & Surveying Eng. Faculty. It is planned n the future to build upon it an e-learning self educating material in form of cd.

The Differential Geometry approach

- The plumb-line is divided in k segments
- The length of each segment $H_i, i = 1, 2, …, k$ is given by the Pythagorian sum

$$H_i = \sqrt{s_{1i}^2 + s_{2i}^2} \,, \quad i = 1, 2, …, k$$

where S_{1i} is measured along the tangent line at the initial point and S_{2i} is the distance of the last point of the segment and the tangent line.

The Integral approach

The length of the plumb-line is given by the formula

$$H_P = \int_0^{\tau_1} \left| \frac{d\,\overline{a}_1}{d\,t} \right| d\,t + \int_0^{\tau_2} \left| \frac{d\,\overline{a}_2}{d\,t} \right| d\,t + … + \int_0^{\tau_k} \left| \frac{d\,\overline{a}_k}{d\,t} \right| d\,t$$

and if $H^{\overline{a}}{}_P$ is the real length of the plumb-line then it holds that

$$\left| H^{\overline{a}}{}_P - H_P \right| < \varepsilon \sum_{i=1}^{k} t_i \,, \ 0 < \varepsilon < 10^{-7}$$

A MATHEMATICAL APPROACH TO DETERMINE THE LENGTH OF THE PLUMB-LINES OF THE EARTH'S GRAVITY FIELD

HELLENIC REPUBLIC
NATIONAL TECHNICAL UNIVERSITY OF ATHENS

ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
**ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ**

G. Manoussakis 　　 P. Tsekrekos
B. Massinas 　　 M. Doufexopoulou

35th COSPAR
Scientific Assembly
Paris, 18 – 25 July 2004

The third objective implemented by the coordinator M. Doufexopoulou and B. Massinas, used theoretical and collected experimental experience on educational approaches and experimental bibliographical research on didactical approaches by Doufexopoulou, in building a Java applet prepared by Massinas, that illustrates the variation of the gravitational potential energy between different surfaces of equal potential W nearby and at distance from Earth. The first product of this research appeared as a paper presented in the educational session of the 35th general assembly of COSPAR:

Teaching orientation analysis in space science education. The case of the Earth's gravity field
 (B. Massinas, M. Doufexopoulou)
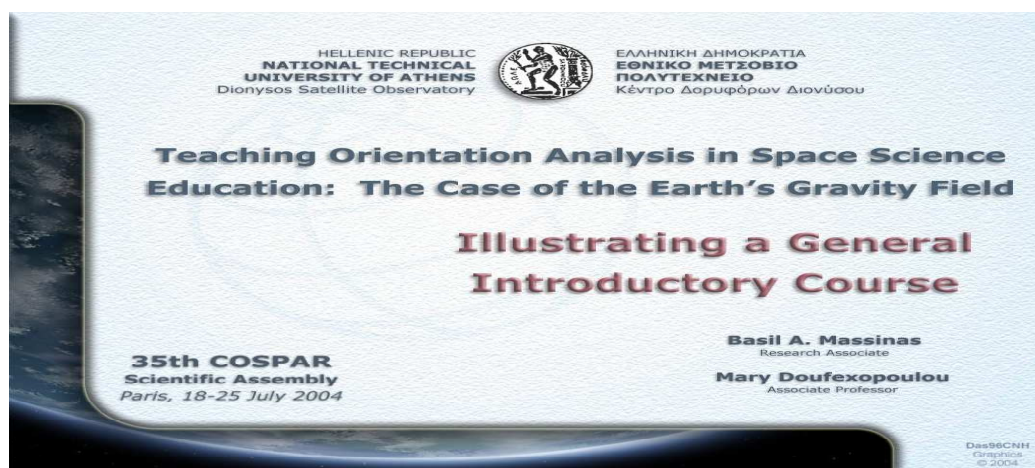
The paper combines didactical rules, the duality of gravity space in concept of geometry and as Newtonian theory and uses the optical representation via Java programming.

Last but not least the frame of this project's research initiated further collaborations in original experimental research in which the difference between the two height systems is treated as an output geo-signal of the unknown Earth's system via spectral analysis of its frequency. First results of this research were presented in the International Association of Geodesy Conference in Jaen, Spain March 2005 in oral presentation titled: Spectral Analysis of Geoidal signals at points of geodynamical interest used in the investigation of the depth of mass-density causal "sources" of ground deformations (Doufexopoulou, M., Bartha, G., Massinas, B).

The paper is submitted for peered reviewing to Springer (in process) and is available by mairiedf@central.ntua.gr or billmass@central.ntua.gr.
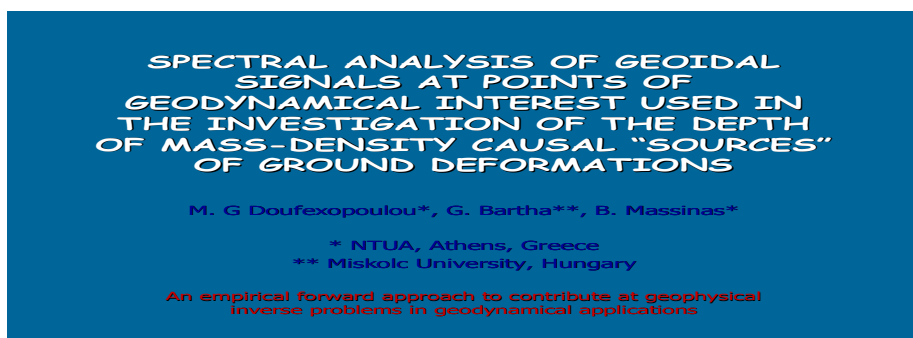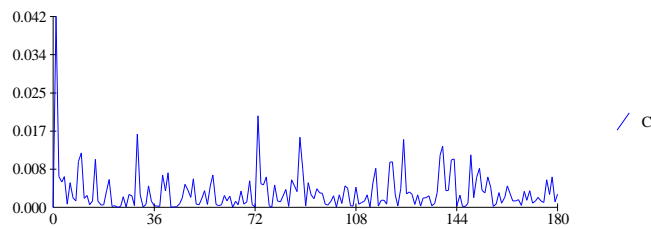
GRACE-27 PowerSpectra

Fig. 1 A sample power spectrum (FFT) from height difference at a point in the vicinity of the Athens earthquake on Sept 7, 1999

A most recent collaboration (April 2005) on educational issues of Geodesy in its Mathematical and Physical foundations (in wider frame than didactics) based on previous know-how of the coordinator and bibliographical search ended up with the joint presentation (with B. Massinas) entiltled: Geodetic Education for the Earth's and Space exploration. Impact of recent trends and practical needs (M.G. Doufexopoulou, Ass. Prof. Physical Geodesy – B. Massinas, Research Associate NTUA, Athens, Greece) Annual Assembly of European Geoscience Union – Vienna 2005. This work was also presented during the works of the 3d General Assembly of the Educational Network EEGECS in Geodesy (Erasmus / Socrates) in Dublin 2005, as contribution of high interest
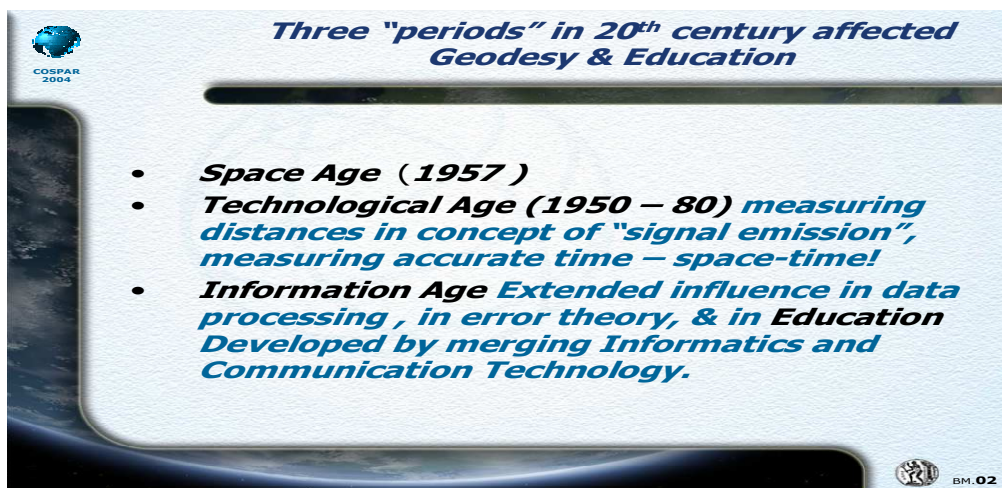
(a sample of page in ppt presentation).

The full text of this paper is submitted as contribution to the Erasmus/Socrates Educational network EEGECS under the Working group of Postgraduate research.

Finally, the present Thales project initiated a profound frequency analysis investigation of the differences between the two types of height in respect to the newest models of the gravitational potential W that are under experimental testing in Europe (submission of a proposal under "Lefkippos" project announced by NTUA with extended research team from other Universities).

Conclusions

The two theoretical approaches to determine the length of a plumb-line revealed the existing built in the problem, difficulties in achieving accuracies that can be of practical use in Geodesy. The rate of the accuracies in both approaches needs to be investigated by programming the formulae and comparing the results with observed heights. Instead the frequency analysis of

point signals constructed from the differences of the two types of heights has shown several new directions of applied and basic research in Earth and space disciplines. Possible directions are the spectral resolution of geo-signals that can be achieved with alternative spectral methods, the continuation of applied research to use the geo-signal analysis as a low cost initial prospecting method (new) and the use of signal analysis as a means to check the accuracy of the harmonic coefficients of global scale models of geopotential W at high degrees of expansion. Last but not least the educational part of this research project revealed new approaches of composing the educational material in subject topics that are not apparently useful in engineering applications although they are fundamental. This conclusion should be considered as indicative only as it is based on a quiz questionnaire of few students (15) for only one semester (2004 -2005) which attended the introductory lecture about the Earth's gravity field.

PUBLICATIONS

1) *"A Mathematical Approach to determine the length of the plumb-lines of the Earth's Gravity field"*
(G. Manoussakis, B. Massinas, P. Tsekrekos, M. Doufexopoulou)
 (The paper accepted for publication by Elsevier in the 'Advances in Space Research').

2) *"Teaching orientation analysis in space science education. The case of the Earth's gravity field"* (B. Massinas, M. Doufexopoulou)
(The paper accepted for publication by Elsevier in the 'Advances in Space Research').

3) *"Spectral Analysis of Geoidal Signals at points of geodynamical interest used in the investigation of the depth of mass-density causal 'sources' of ground deformations* (M. Doufexopoulou, G. Bartha, B. Massinas) (**submitted for publication in Springer**).

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