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The Inverse Scattering Problem from Infinite Surfaces – Location of Buried bodies

Research Team

Kiriakie Kiriaki (P.I.), *Professor,**
Drossos Gintides, Lecturer *PD 407, *, Lecturer, Hellenic Naval Academy*
Nikolaos Kavallaris, *Postdoctoral Researcher, **
Konstantinos Anestopoulos, *Doctoral candidate,**
**School of Mathematical & Physical Sciences, National Technical University of Athens, Greece*
Antonios Charalambopoulos, Professor, University of Ioannina, Greece
Andreas Kirsch, *University of Karlsruhe, Germany*

Introduction

In this research project we have studied the inverse scattering problem from infinite surfaces and the location of buried bodies. First we studied the well possedness of the direct scattering problem for cavities, scatterers which satisfy Dirichlet boundary conditions or / and inclusions with different elastic properties from the surrounding medium. We considered also the scattering from infinite periodic elastic structures exploiting Ewalds representation for the dyadic Green's function. The successful approach of the direct problem gave us the opportunity to face the inverse problem. We have used the "factorization" method, proposed by Professor A. Kirsch in acoustics. We focused in two different cases. In the first case we have considered inclusions with different elastic constants and without absorbsion. In the second case we have studied the problem of locating and determining the geometrical properties of a scatterer totally imbedded in an elastic medium defined from an infinite surface.
Algorithms have been proposed and numerical results have been elaborated justifying completely the theoretical considerations. All the above results are very important for many applications.

A. Project objectives

The purpose of this research work was the study of the inverse scattering problem form infinite surfaces and the location of buried objects. The problem of determining their shape was based on the knowledge of the scattered field in the far field region or near the scatterer.

B. Direct Scattering Problem

It is well known that the direct scattering problem is the determination of the scattered field in case where a body, the scatterer, disturbs the propagation of a wave field. The scattering process from infinite surfaces needs a careful treatment in order to secure the well possedness of the corresponding mathematical description.

In the starting step the research team considered the simpler problem for the scattering process from bounded domains. This approach was necessary to gain the experience in the area of linear elasticity.
We faced the following problems:

- The well possedness for problems concerning cavities, bodies with Dirichlet conditions or inclusions.
- The linear superposition principle and the properties of the interior eigenvalues. The application of these arguments guarantees the existence of the approximate solvability of the far-field or near field equations for a superposition which corresponds to a point source located inside the scatterer.
- The scattering from infinite periodic elastic structures evaluating the representation method due to Ewald. This method can be used to assure well possedness for direct problems regarding periodic structures and has the very important numerical behavior of fast convergence. Results of this work are presented in [5].

C. The Inverse Scattering Problem
The treatment of inverse problems is a very difficult subject and this is due to the fact that these problems are non linear and ill posed. The inverse problem that we considered was the location of the scatterer and the determination of its geometrical properties.
The research team has applied the linear “sampling” method proposed by D. Colton in acoustics and electromagnetism. The main algorithm is based on numerical approximation of the far field equation. We have applied this method to recover the shape of many different cases of scatterers. Results of this work are presented in [1,2,3 και 4].

D. Numerical Algorithm
<u>Direct Problem</u>
The numerical treatment of the direct problem was based on the method of integral equations. The numerical approximation was based on Nystrom’s method taking into account the special features of each special case for example the strong singularities of the integrals in elasticity.
<u>Inverse Problem</u>
One of the main tasks of this project was the implementation of fast algorithms appropriate to solve numerically the reconstruction of scatterers in elastic media.
We have adopted the linear sampling and factorization methods and we have developed very fast algorithms to solve the inverse problems (see fig.1 and fig.2). It is characteristic to underline that the computation time is less than the time needed to solve the direct problem. Furthermore, no a priori information about the locations or the geometry is necessary.

Applications
The results of this project could be used in a wide range of applications:
Detection of buried objects, non destructive evaluation of materials,
investigation of the see environment and the bottom also, medical applications etc.

Conclusions

This work contributed scientifically in the scattering theory from infinite surfaces and in the location and determination of the shape of buried bodies. The approach was based on novel reconstruction methods which lead to fast computational algorithms.

The results are of a high degree of scientific and technological excellence and could be applied in many applications.

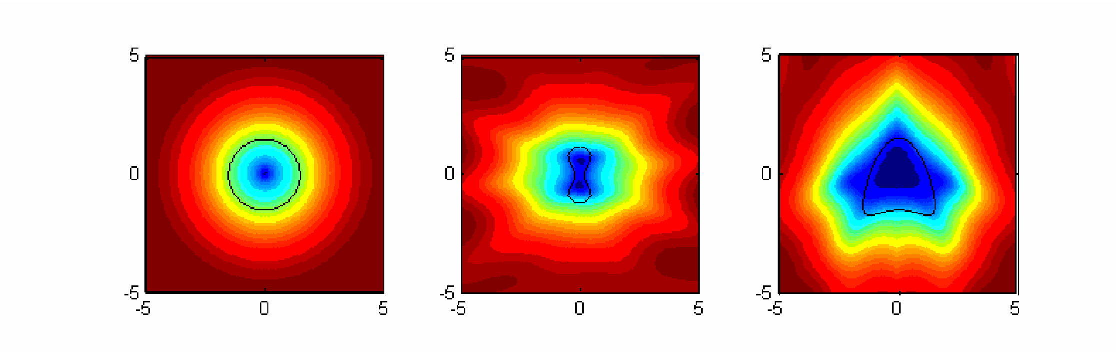

Figure 1 Reconstructions with 50% P and 50% S-waves. The original shapes are indicated with a black line.

Numerical results for the inverse scattering from two spherical inclusions changing the distance between them

Parameters:
Isotropic medium
Right scatterer: radius a=1.2, k_p 2a=2.19, k_s 2a=4
Left scatterer radius b=1.2, k_p 2b=2.19, k_s 2b=4

(a)
distance between the scatterers d=12.67

(b)
distance between the scatterers d=5.14

(a)
distance between the scatterers d=4.19

(b)
distance between the scatterers d=0.01

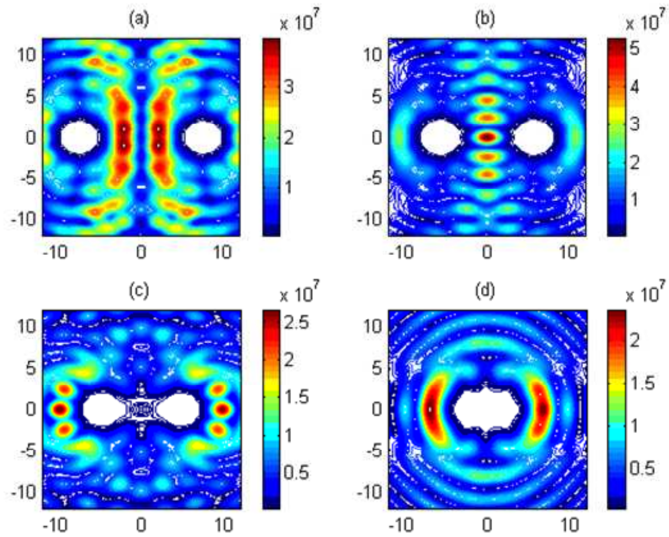

Figure 2 Examples of reconstructions in linear elasticity using the linear sampling method.

References

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Conferences

1. *6th* Workshop on Applied Mathematics in Science and Modern Technology, Tsepelovo, 18 – 21 September, 2003.
2. International Conference of Influence of Traditional Mathematics and Mechanics on Modern Science and Technology , Messini, 24-28 May 2004.