THALES Project No. 1179

Solving Mixed Integer and Nonlinear Programming Problems by Genetic Algorithms

Research Team

Haralambos Sarimveis (P.I.), *Lecturer, NTUA, Greece*

Alex Alexandridis *(Research Student), NTUA, Greece*

Athanasios Nikolakopoulos *(Research Student), NTUA, Greece*

Antonios Spyropoulos *(Post-Doc Fellow), NTUA, Greece*

Introduction

The objective of this research program was the development of methodologies for the solution of Mixed Integer and Linear Programming (MINLP) problems, based on the utilization of genetic algorithms. The main result was the development of a hybrid method that combines the advantages of genetic algorithms with other evolutionary and traditional optimization techniques. Although the methodology that has been developed is of general use, the research effort was concentrated on the solution of production planning and model predictive control problems, which are of great interest for the industry.

Modeling systems as MINLP problems

The development of MINLP models that describe complex systems is essential for the application of an optimization algorithm and the measurement of its performance. Two different systems were modeled as MINLP problems in this work:

a) The production scheduling system in a lube production plant. The proposed methodology splits the time horizon into several periods and formulates an optimization problem, where detailed mass balances describe the relationships between the different process variables [1]. The discrete time representation was selected due to the fact that the scheduling horizon is usually rather long (more than 5 days), so that a continuous time model could be of lower practical applicability. The objective function is formulated to describe in a realistic manner the total cost throughout the time horizon. The model takes into account special requirements of the lube production plant that are not considered by most generic models. For example, it can handle special storage limitations and can generate a schedule, where the products are not produced in the same order in all stages.

b) An adaptive control system based on the prediction of a dynamic discrete-time adaptive model. This system requires at each time instance the solution of a nonlinear optimization problem, aiming at the determination of the optimal control moves [2,3]. The main difficulty in the solution of this particular problem is the inclusion of the "persistent excitation" criterion which is formulated as a matrix function of the control moves that must be positive definite. Inclusion of this additional constraint renders the optimization problem non-convex, meaning that standard optimization methods require long computational times that are not acceptable for a problem that must be solved in real time.

Development of an evolutionary algorithm for the solution of Nonlinear Programming (NLP) problems.

A complete framework was presented [4] for solving nonlinear constrained optimization problems, based on the line-up differential evolution (LUDE) algorithm which was proposed for solving unconstrained problems. The LUDE algorithm is an iterative stochastic methodology that starts with a random population of possible solutions. The fitness of each solution is measured by computing the corresponding value of the objective function. Then new generations are produced by lining up the solutions according to their fitness and applying the LUDE crossover and mutation operators. The positions of the solutions are very important, since they determine to what extent the crossover and mutation operators are applied to each solution. Constraints are handled by embodying them in an augmented Lagrangian function, where the penalty parameters and multipliers are adapted as the execution of the algorithm proceeds:

$$F(\mathbf{x},\mathbf{a},\mathbf{b},\mathbf{p},\mathbf{q}) = f(\mathbf{x}) + \sum_{m=1}^{M} a_m \left[(h_m(\mathbf{x}) + p_m)^2 - p_m^2 \right] + \sum_{k=1}^{K} b_k \left[\langle g_k(\mathbf{x}) + q_k \rangle_+^2 - q_k^2 \right] \qquad (1)$$

where a_m and b_k are positive penalty parameters, and the corresponding Lagrange multipliers p_m and q_k are positive as well. In the above equation $f(\mathbf{x})$ is the objective function to be minimized, while $h_m(\mathbf{x}) = 0,\ l = 1,...,M$ and $g_k(\mathbf{x}) \geq 0,\ k = 1,...,K$ are the equality and inequality constraints respectively.

The efficiency of the proposed framework was illustrated by solving numerous optimization problems and comparing the results with those obtained by other evolutionary techniques that can be found in the literature. Some of the results and comparisons with other techniques are presented in Table 1, where the superiority of the proposed algorithm is illustrated. The complete optimization problems are not shown here due to space limitation but can be found in Ref. [4].

Development of an evolutionary algorithm for the solution of MINLP problems.

In its most generic formulation, an MINLP problem can be described as a mathematical system, consisting of a nonlinear objective function and a number of equality or inequality constraints, where both continuous and integer variables are involved. If we assign specific values to the integer variables, the problem is transformed to an NLP problem, where the integer variables are no longer decision variables.

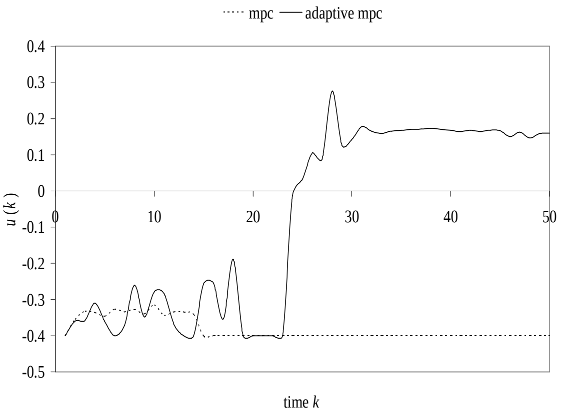
Ex.. #		Exact Solution	Proposed Algorithm	HHE-MUM-APP	TPEP	Hybrid EP	EP alone
1	*fmin*	0.25000	0.25000	0.25000	0.25000	0.83636	0.24510
	Er	0	0.0	0.0	0.0	Diverged	0.0049
	N	-	2250*	796*	4658	7667	5014
2	*fmin*	-5.50801	-5.50801	-5.50801	-5.50801	-5.42623	-5.51096
	Er	0	0.0	$1.14 \cdot 10^{-9}$	0.0	0.08178	diverged
	N	-	41400*	52615*	3659	8081	4392
3	*fmin*	-6961.81388	-6961.81388	-6961.81388	-6961.81388	Diverged	1033.13591
	Er	0	0.0	$7.9 \cdot 10^{-9}$	0.0	Diverged	diverged
	N	-	78450*	223455*	21739	Diverged	14049
4	*fmin*	5.00000	5.00000	5.00000	5.00000	5.00000	4.99800
	Er	0	$3.0 \cdot 10^{-12}$	0.0	0.0	Diverged	0.002
	N	-	31200*	32343*	19139	13130	78851
5	*fmin*	1.00000	1.00000	1.00000	1.00000	Diverged	0.99559
	Er	0	0.0	0.0	0.0	Diverged	0.00441
	N	-	48150*	123191*	3730	Diverged	4251
6	*fmin*	0.05400	0.05400	0.05395	0.05400	Diverged	0.43321
	Er	0	$1.8 \cdot 10^{-13}$	$1.5 \cdot 10^{-9}$	$2.15 \cdot 10^{-8}$	Diverged	diverged
	N	-	125700*	271023*	11210	Diverged	1448

Table 2. Comparative results for a number of benchmark problems

The main goal of the proposed algorithm is the efficient selection of values for the integer variables. The algorithm has been named “GATSA” as an acronym of the words Genetic Algorithms, Tabu Search and Simulated Annealing, since we have adopted characteristics from all three stochastic methods [5]. Each selection of values for the integer variables is accompanied by the solution of the nonlinear problem that is formulated. When a solution is obtained, the algorithm first checks if it satisfies any of the termination criteria. If not, the procedure is repeated by mutating the current solution using standard operators. These are based on the random selection of some of the integer values and the modification of their values. According to the specific problem, we may give a different possibility of selection to each possible value, which may be adapted as the algorithm proceeds. In order to guarantee that the integer vector that is selected for the next iteration is not coinciding with any of the integer vectors that have been used so far, the algorithm retains a tabu list. Ideas from the simulating annealing method have been adopted, by giving the algorithm the ability to accept a solution even if it does not modify the objective function towards the desired direction. The possibility of accepting such a solution is reduced, as the algorithm proceeds. The method was applied with success to the optimal design of chemical processes and the results can be found in [5].

Application of the new stochastic methods on the solution of large-scale optimization problems

a) The LUDE algorithm was applied for the solution of complex non-convex model predictive control problems. The complexity was increased, by formulating multiobjective optimization problems. One of the goals was to test the efficiency of the MPC method cases where sudden changes in the dynamic behavior are observed. Two examples were tested. The first concerns a single input singe output (SISO) system, while the second is a multi input multi output (MIMO) system with two input and two output variables. In both systems we initially change the set points of the controlled variables. At a specific time instance a sudden change in a system parameter occurs and the algorithm is asked to keep the controlled variables as close as possible to the desired values. The first objective to be satisfied is the persistent excitation condition, and next the proposed framework minimizes the rest of the objective functions, which are positioned in a priority list according to their importance.



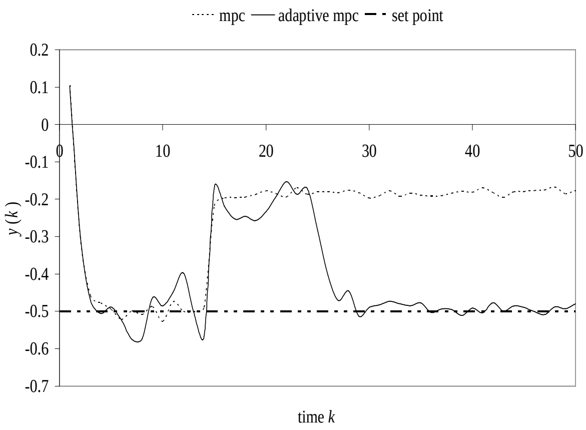


Figure 1. Responses of the input and output variables in the first example, where a sudden change in the dynamics of the system happens in time instance 15. Comparison with the standard MPC methodology.

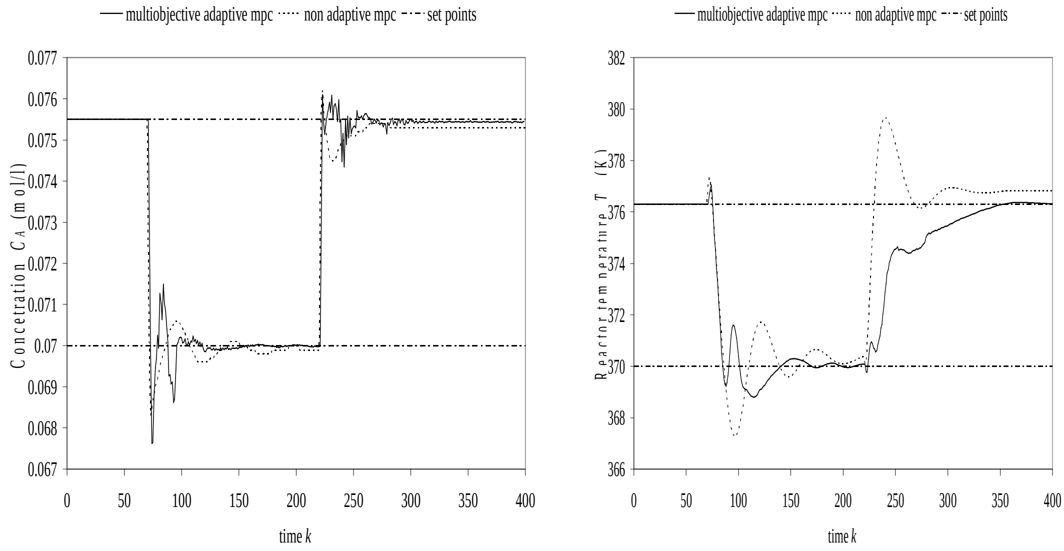


Figure 2. Responses of the two controlled variables in the second example. Comparison with the standard MPC methodology.

In both examples it is shown that the utilization of the proposed evolutionary algorithm for the solution of the optimization problems is successful, since it manages to determine the values of the manipulated variables that can drive the system to the desired set point. It should be mentioned that the traditional optimization techniques that were tried for the solution of the same problems did not produce successful results in acceptable times.

b) The hybrid algorithm that was developed for solving problems where continuous and integer variables are involved, was used for finding the optimal schedule in a lube production plant. The efficiency of the algorithm gave us the ability to increase the number of periods in which the future production horizon is partitioned, by increasing proportionally the number of binary variables. This resulted in the calculation of more flexible schedules that improved significantly the value of the objective function. As an example, the following table is shown that presents the solution to the same scheduling problem, using 6,12 and 24 periods.

Sales (tn)	6 periods	12 periods	24 periods
1	0.0	0.0	0.0
2	607.8	607.8	607.8
3	1646.4	1636.3	1636.3
4	507.0	507.0	536.3
5	0.0	308.0	132.5
6	616.0	616.0	770.0
Total profit(€)	768,935	1,062,161	1,076,600

Table 2. Results on the optimal lube scheduling problems using different time partitions.

Conclusions

During this project new stochastic methodologies were proposed for solving NLP and MINLP problems. The methodologies were tested on both benchmark problems that can be found in the literature and large-scale problems. They proved successful with respect to both accuracy and required time for convergence. Thus the methods that were developed can be used as robust and reliable approaches for facing non-convex and/or non-continuous constrained optimization problems, which are difficult to solve by traditional optimization algorithms.

References

1. Doganis Ph., Sarimveis, H., Bafas G. and Koufos D., "Production optimization in a lube production plant using mathematical programming", 4th Panhellenic Chemical Engineering Conference, Patras, Greece, 2003.
2. Alexandridis A., Sarimveis H. and Bafas G., "Modeling of continuous digesters using adaptive RBF neural network models", 11th Mediterranean Conference on Control and Automation MED'03, Rhodes, Greece, 2003.
3. Alexandridis A., Sarimveis H. and Bafas G., "Adaptive control of continuous pulp digesters based on radial basis function neural network models" ESCAPE 13, Lappeenranta, Finland, 2003.
4. Sarimveis H. and Nikolakopoulos A., "A line up evolutionary algorithm for solving nonlinear constrained optimization problems", *Computers and Operations Research*, **32(6)**, 1499-1514, 2005.
5. Nikolakopoulos A, Sarimveis H and Bafas G, "Development of a heuristic algorithm for the solution of MINLP problems with application in the optimization of chemical engineering systems", 4th Panhellenic Chemical Engineering Conference, Patras, Greece, 2003.