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The nonlinear dynamics of the coupled ship rolling in steep beam waves

Research Team

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1. Introduction

The fear of capsizing of a ship in waves is one of the most basic concerns for mariners, passengers and for anyone with a vested interest on maritime transportation. As a matter of fact, the development of deeper understanding about the dynamics of ship rolling motion and the factors that affect ship stability should be of crucial importance for the maritime community. The primary objective of our research is the modeling of the coupled ship rolling motion in steep beam waves and the development of a reliable procedure for the assessment of stability of intact ships, taking into account the probabilistic character of the wave environment where ships operate.

Conditions that can lead to capsize are summarized, for example, in the book of Belenky & Sevastianov [1]. Typically, for beam seas, large amplitude rolling motion can occur due to resonance, i.e. when the incident wave frequency is close to the natural roll frequency of the ship. Other mechanisms that can generate instability are: shift of cargo due to large angular accelerations; the shipment of green water on the deck which undermines static stability and also induces sloshing (Grochowalski et al [2]); and finally, impact loads due to breaking waves on the ship's side (Ishida et al [3]). Breaking waves are more likely to affect the stability of small ships like fishing vessels.

It is well known that excitations with these origins may cause nonlinear dynamic behaviour and, in fact, nonlinearity may have significant influence on the propensity of a ship for capsize. The nonlinear dynamics of rolling motion has been examined thoroughly in the past, but only under the assumption that one or two degrees of freedom can capture sufficiently system response. For a detailed review see Spyrou & Thompson [4], Thompson [5] and Falzarano et al [6]. The coexistence of more than one roll responses for the same parameter values of the system and the possibility of dangerous jumps of the response from small to large amplitude as some parameter (like the wave frequency) is varied, are well-known manifestations of strongly nonlinear rolling, predicted from theory and confirmed from experiments (Cotton & Spyrou [7]). The prediction of these phenomena with more detailed models of ship hydrodynamics and the development of effective stability assessment methodologies are the current trend at international level and represent the direction pursued by our research.

2. Objectives

- To build a detailed mathematical model for the coupled heave, sway and roll motions of a ship in beam regular waves that can be used up to capsize limits.

- To take a few steps towards determining factors that affect capsize propensity, using the mathematical model, and to determine the characteristics of critical wave encounters.
- To integrate the "deterministic" tools developed earlier within a stability assessment method that accounts for the stochastic nature of the sea environment.

3. Limitations of single-degree-of freedom models: the effective gravitational field

The mathematical model of rolling motion of one degree-of-freedom is based on the assumption that the ship tends to follow the circular motion of the wave particles and so makes use of the effective gravitational field concept; that is, the gravity force is always perpendicular to the instantaneous water surface. This approach has originated from Froude. This concept is based on the assumption of long incident wave relatively to the beam of the ship. As we shall prove, this assumption has limitation as the wave steepness is increased. Furthermore, if our interest lies in waves with length comparable to the ship's beam, the method deviates and may not be fully representative of the physical system.

When a body follows the circular motion of wave particles, a time varied gravitational acceleration $g_e(t)$ act on it, which is also perpendicular to the wave slope. According to figure 1, the condition of perpendicularity is expressed as follows:

$$\gamma = \beta - n_y = 0 \tag{1}$$

where γ is the angle that characterizes the error from perpendicularity, n_y is the wave slope and the angle β is calculated as follows:

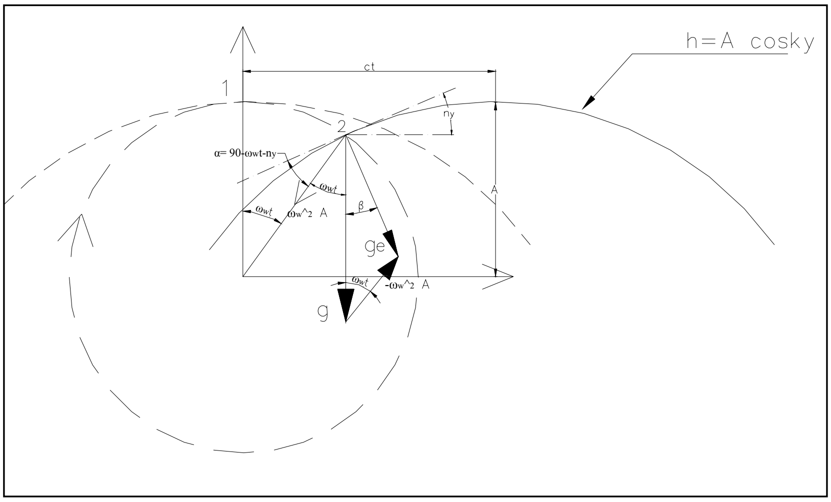

Figure 1. The rotational motion of a wave particle in deep water and the forces acted on it.

$$\beta = \arcsin\left(\frac{\omega^2{}_w A\sin(\omega_w t)}{\sqrt{g^2 + A^2\omega^4{}_w - 2gA\omega^2{}_w\cos(\omega_w t)}}\right) \tag{2}$$

$$\eta_y = \mathrm{A}\frac{\omega^2{}_w}{g}\sin(\omega_w t) \tag{3}$$

$$g_e = g - A\omega^2_w \cos(\omega_w t) \quad \textbf{(4)}$$

where A and ω_w are the wave amplitude and frequency.

Figure 2 shows the variation of the angle γ (scaled over the wave slope Ak) with the wave steepness H/λ and the ratio λ/B, where k, λ are, respectively, the wave number and length. B is the beam of the ship. From the above figure it is clear that the assumption of perpendicularity is not valid when wave steepness is high (above 1/20) and when the wave length is shorter than five times the beam of the ship (customarily, the long wave assumption refers to a wave that is longer by at least six times to the ship beam).

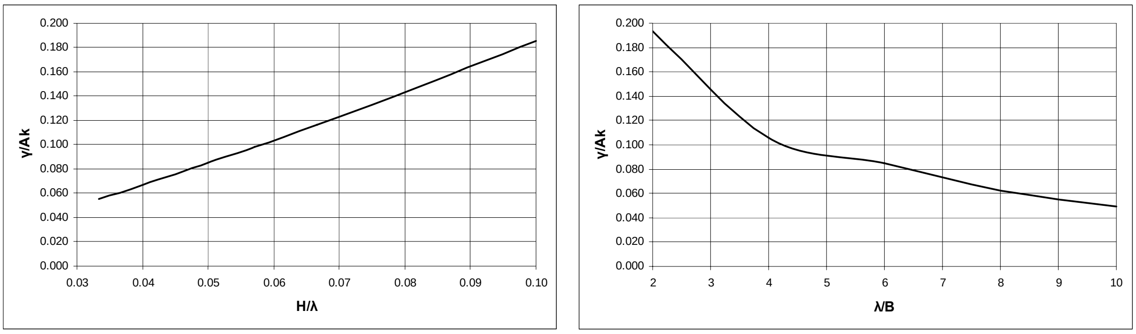

Figure 2. The effect of wave steepness and wave length.

4. The mathematical model of the coupled rolling motion

The basic equations of motions for the sway, heave and roll directions are expressed as follows:

$$m(\dot{v} - \dot{\phi}w) = \sum F_y \quad \textbf{(5)}$$

$$m(\dot{w} + \dot{\phi}v) = \sum F_z \quad \textbf{(6)}$$

$$I_G \ddot{\phi} = \sum M_G \quad \textbf{(7)}$$

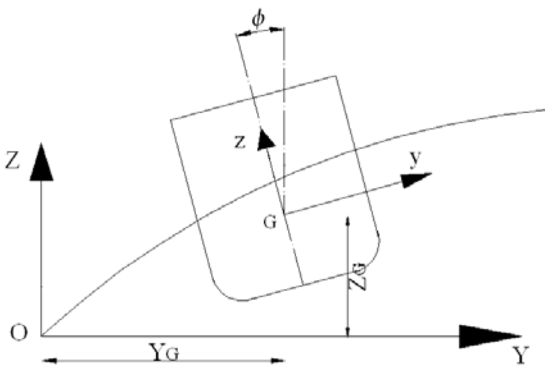

Figure 3. The ship in waves. The earth and body fixed coordinates system.

where v, w and $\dot{\phi}$ are the sway, heave and roll velocity of the centre of ship's gravity, m and I_G its mass and moment of inertia about the roll axis, whereas $\sum F$ are generally the total forces (and moment) that act on the ship and more analytically

$$\sum F = F_{Hs} + F_W{}^{FK} + F_W{}^{D} + F_R + F_V \quad \textbf{(8)}$$

where F_{Hs} the hydrostatic forces, F_W^{FK} the Froude – Krylov forces, F_W^D the diffraction forces, F_R the radiation forces και F_V the viscous forces. To calculate the forces and solve numerically the system of differential equations we have used the program Mathematica. Input data are as follows: concerning the ship, the geometry of the hull, mass and distribution of mass. Concerning the incident wave, its height and frequency. The code creates panels over the hull where the static and dynamic pressures are calculated from the instantaneous wave elevation z_w for any time step as well as the angle β between the horizontal plane and the normal vector of the panel as the next figure shows.

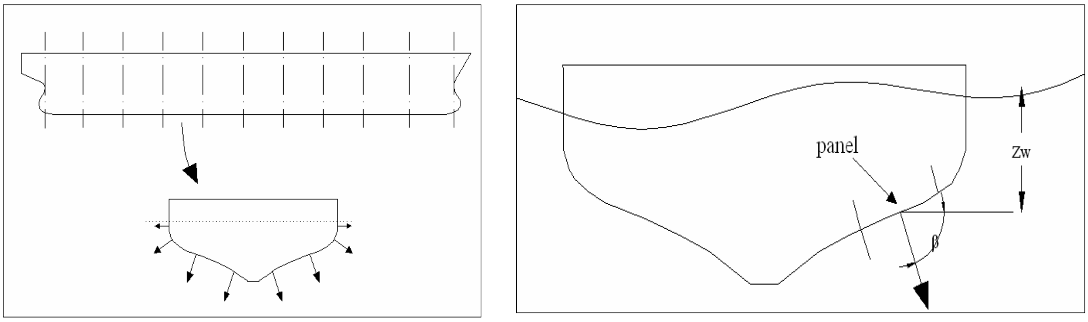

Figure 4. Creation of panels

With integration of the static and dynamic pressures on the instantaneous wetted surface, the nonlinear hydrostatic and nonlinear Froude – Krylov forces are calculated. It is noted that it is important to include the nonlinear part of the forces because it is influential for the large amplitude rolling motion. The second order Froude – Krylov "drift" force is included. Moreover, the deck-in-water effect is taken into account. The viscous forces, including roll damping, cross coupling forces between sway and roll and sway drag force, are estimated by calculating separately each component of damping force, like eddy – making and bilge keel damping, using the relative local velocities and the detailed geometry of the hull. Although radiation forces have been modeled solving the convolution integrals of memory effects in time domain, in the present model they are calculated through the constant coefficients of added mass and damping. In the next stage the diffraction forces are going to be estimated also.

5. Application of the mathematical model: investigation of dynamic stability

As application we have used a Japanese fishing vessel whose body plan and roll restoring characteristics are shown in figure 5. Numerical simulations of the response of this ship (decay test and response close to resonant beam waves) based on our new mathematical model, are shown in figure 6. In figure 7 we see how wave steepness affects the amplitude of rolling.

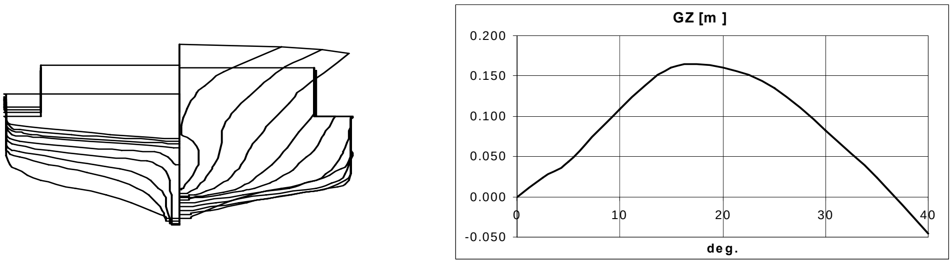

Figure 5. Body plan and the GZ curve in calm water calculated by the numerical code.

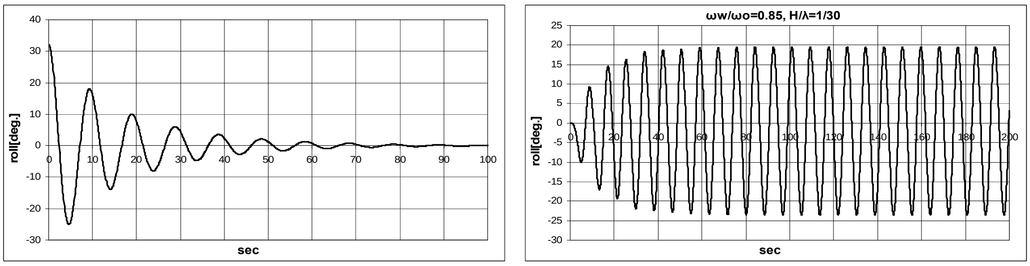

Figure 6. Time domain simulations of roll angle for free decay from "large" angle and for beam seas with frequency close to natural roll frequency.

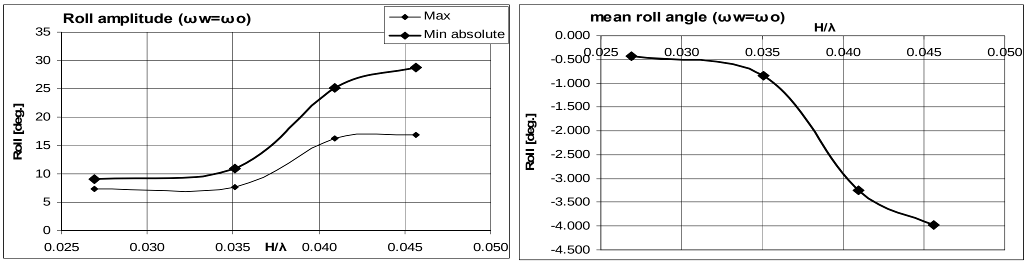

Figure 7. The effect of wave steepness on roll amplitude and mean roll angle.

6. Assessment procedure

The developed mathematical model can be used as a tool of an assessment methodology of ship stability. For easier comprehension we shall demonstrate it through an example. For a more detailed presentation see Spyrou [8]. Assume that, for our fishing vessel, it is considered as unacceptable to exceed a roll angle of 20^0. By using the numerical code we can determine the critical wave group encounter, in terms of wave height, period and run length. Assume that seven wave crests above a critical height H_{cr} = 3.973 m and with a period T_w=8.7 sec are sufficient for causing excessive rolling. We estimate the probability to encounter a wave group with these characteristics. The probability can be determined by using available in the literature joint distributions of wave height and period $p(H, T)$ or joint distributions of successive wave heights $p(H_1,H_2)$, see for example Longuet - Higgins [9], Kimura [10] and Tayfun [11]. We would prefer to use joint distributions of successive wave heights and periods as these are more suitable for our approach. Such distributions are now under consideration. Here we proceed with the distribution of successive wave heights. The probability to encounter a wave group with two successive wave heights above a critical wave height will be:

$$P[H_1 \geq H_{cr} / H_2 \geq H_{cr}] = \int_{H_{cr}}^{\infty} \int_{H_{cr}}^{\infty} p_{HH}(H_1, H_2) dH_1 dH_2 \Bigg/ \int_{Hcr}^{\infty} p_H(H) dH \qquad \textbf{(9)}$$

where $p_H(H)$ and $p_{HH}(H_1, H_2)$ are the Rayleigh and the bivariate Rayleigh distribution (see Kimura [10] and Tayfun [11]). For example, using the I.T.T.C spectrum with significant height H_S=3 m, we find that $P[H_1>H_{cr}/H_2>H_{cr}]$ =0.103. We can reasonably assume that the wave height H_{n+1} depends only on H_n and not on $H_{n-\kappa}$, κ=1, … ("Markov chain"). Then

$$P_{H_1 \ldots H_j} = P_{H_1 H_2}^{j-1}(1 - P_{H_1 H_2}),\ \text{j=7} \tag{10}$$

And the probability of encounter of the critical wave group is: $P[H_1>H_{cr}/H_2>H_{cr},/...,H_7>H_{cr}] = 1.06*10^{-6}$.

7. Concluding remarks

We have developed a detailed mathematical model of ship rolling in beam waves that takes into account the coupling with heave and sway motions. We are currently working on improving this model by implementing the calculation of diffraction force and also the effect of breaking waves. The model is used as a tool in a newly developed assessment procedure that takes into account the probabilistic character of sea waves. The use of analytical and geometrical techniques on this model has been planned for a more rigorous assessment.

8. References

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